Euler formula & Euler identity

Rabu, 09 Desember 2020 18.18

$f(x) = \cos x$
$f'(x) = -\sin x$
$f''(x) = -\cos x$
$f^3(x) = \sin x$
$f^4(x) = \cos x$

when $x = 0$
$f(0) = 1$
$f'(0) = 0$
$f''(0) = -1$
$f^3(0) = 0$
$f^4(0) = 1$

we can see the pattern
1, 0, -1, 0, 1, 0, -1, 0, 1...

$f(x) = \cos x = 1 + 0x - \frac{1}{2!}x^2 + 0 + \frac{1}{4}x^4 \ldots$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + \ldots$$

$f(x) = \sin x$
$f'(x) = \cos x$
$f''(x) = -\sin x$
$f'''(x) = -\cos x$
...

while $x = 90°$

$f(x) = 0$
$f'(x) = 1$
$f''(x) = 0$
$f'''(x) = -1$
$f^4(x) = 0$
$f^5(x) = 1$
...

Maclaurin Series of sin x

$\sin x = 0 + x + 0 - \frac{1}{3!}x^3 + 0 + \frac{1}{5!}x^5 \ldots$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 - \ldots +$$

$f(x) = e^x$
$f'(x) = e^x$
...
$f'(0) = 1$
$f''(0) = 1$
...

Maclaurin Series for Euler Number

$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 \ldots$

$e^{ix} = 1 + ix - \frac{1}{2!}x^2 - i\frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 \ldots$

$e^{ix} = \left(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \ldots\right) + i\left(x - \frac{1}{3!}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \ldots\right)$

cos x term — sin x term

So, $e^{ix} = \cos x + i \sin x$ ← euler formula

now, we know that $e^{ix} = \cos x + i \sin x$
what happen if $x = \pi$
$\cos \pi = -1$
$\sin \pi = 0$

$e^{i\pi} = -1 + i \cdot 0$

$e^{i\pi} = -1$ ← euler identity

© Fikri Mulyana Setiawan